

TEST 3A

Name: _____

Instructions:

- The exam consists of four (4) problems, some of which may have several parts. It has four (4) pages (including this one); you should make sure that you have all of them before you start.
- Turn off your cell phone or any communications device (if you have one) and put it away, and remove any headphones before beginning the test.
- Show all work in detail or your answer will not receive ANY credit. Write neatly and box all answers.
- Make sure you read the problems and answer everything that is asked. If you are asked to use a particular method, you must use that method to receive full credit. If you are not told to use any particular method, you may use any method mentioned in class.
- No calculators with Qwerty keyboards or ones like the Casio FX-2, TI-89, or TI-92 that do symbolic algebra may be used. If you use your calculator for a calculation, make sure you indicate which expression you are entering into your calculator; do NOT just give a final answer.

Honor Statement: By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or Mathematics Department instructor.

Signature

(1) Do the following for the data points $(-1, -1), (1, 1), (2, 5), (3, 11)$.

(a) [10 points] Find the parabola passing through the origin ($y = ax^2 + bx$) which best approximates the data.

(b) [10 points] Find the parabola symmetric with respect to the y -axis ($y = ax^2 + c$) which best approximates the data.

(2) [20 points] Find the least square solution to the following system of linear equations.

$$\begin{aligned}x_1 &+ x_3 = 1 \\-3x_1 + 3x_2 - 3x_3 &= 2 \\-3x_1 - 2x_2 + x_3 &= 2 \\-2x_1 + x_2 - 2x_3 &= -3 \\-2x_1 + 2x_2 + 3x_3 &= 1\end{aligned}$$

(3) [15 points] Find an orthogonal basis for the subspace spanned by the following vectors.

$$\vec{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 3 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

(4) Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$, and W be the subspace spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

(a) [10 points] Show that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is an orthogonal set.

(b) [15 points] Find the vector which is in W that is closest to $\vec{u} = \begin{bmatrix} -4 \\ 2 \\ -2 \\ -4 \end{bmatrix}$.

(c) [15 points] Find an orthonormal basis for W .

TEST 3B

Name: _____

Instructions:

- The exam consists of four (4) problems, some of which may have several parts. It has four (4) pages (including this one); you should make sure that you have all of them before you start.
- Turn off your cell phone or any communications device (if you have one) and put it away, and remove any headphones before beginning the test.
- Show all work in detail or your answer will not receive ANY credit. Write neatly and box all answers.
- Make sure you read the problems and answer everything that is asked. If you are asked to use a particular method, you must use that method to receive full credit. If you are not told to use any particular method, you may use any method mentioned in class.
- No calculators with Qwerty keyboards or ones like the Casio FX-2, TI-89, or TI-92 that do symbolic algebra may be used. If you use your calculator for a calculation, make sure you indicate which expression you are entering into your calculator; do NOT just give a final answer.

Honor Statement: By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the Mathematics Department and your instructor. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any testing center proctor or Mathematics Department instructor.

Signature

(1) Do the following for the data points $(-1, -5), (0, -4), (2, 4), (3, 11)$.

(a) [10 points] Find the parabola passing through the origin ($y = ax^2 + bx$) which best approximates the data.

(b) [10 points] Find the parabola symmetric with respect to the y -axis ($y = ax^2 + c$) which best approximates the data.

(2) [20 points] Find the least square solution to the following system of linear equations.

$$\begin{aligned}2x_1 + x_2 - 3x_3 &= -3 \\2x_1 + 2x_2 + 2x_3 &= -3 \\3x_1 + 3x_2 - 2x_3 &= 1 \\-3x_1 - x_2 - x_3 &= 3 \\-2x_1 - 2x_2 - x_3 &= 0\end{aligned}$$

(3) [15 points] Find an orthogonal basis for the subspace spanned by the following vectors.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 3 \\ 3 \\ -1 \\ 2 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} 1 \\ -4 \\ 0 \\ 2 \end{bmatrix}$$

(4) Let $\vec{v}_1 = \begin{bmatrix} 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0 \\ -1 \\ -1 \\ 1 \end{bmatrix}$, and W be the subspace spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

(a) [10 points] Show that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is an orthogonal set.

(b) [15 points] Find the vector which is in W that is closest to $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$.

(c) [15 points] Find an orthonormal basis for W .