C. HECKMAN 242
SPRING 2010 FINAL

(1) Let $A = \begin{bmatrix} -7 & -9 & 18 \\ 0 & 2 & 0 \\ -3 & -3 & 8 \end{bmatrix}$. The eigenvalues of A are 2 (with multiplicity 2) and -1 (with multiplicity 1). (You do not need to show this.)

(a) [10 points] Find bases for the eigenspaces of **both** eigenvalues.

(b) [10 points] Is A diagonalizable? If not, explain carefully why not. If so, find matrices D and P, with D being a diagonal matrix, such that $A = PDP^{-1}$.

(2) [15 points] For which values of (x, y) is the following matrix equation true? (Remember that $A^2 = A \cdot A$.)

$$\begin{bmatrix} x & y \\ 3 & 4 \end{bmatrix}^2 + \begin{bmatrix} x & 3 \\ y & 4 \end{bmatrix}^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

(3) [15 points] For which values of x is the determinant of $\begin{bmatrix} x & 0 & 1 \\ 0 & x & 1 \\ x & 2 & x \end{bmatrix}$ equal to zero?

(4) [15 points] Find the orthogonal projection of $\begin{bmatrix} -7 \\ 4 \\ 8 \\ 11 \end{bmatrix}$ onto the subspace spanned by

$\vec{v}_1 = \begin{bmatrix} 9 \\ 2 \\ -6 \\ -2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}$.

(5) [15 points] Find a non-trivial linear combination of $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v}_3 =$

$\begin{bmatrix} 3 \\ 4 \\ -1 \\ -4 \end{bmatrix}$, and $\vec{v}_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -2 \end{bmatrix}$ that adds up to the zero vector.

(6) Consider the following set of data points: $(1, 2), (2, 3), (3, 5), (4, 7), (5, 11)$.

(a) [10 points] Find the line $y = mx + b$ which best approximates the data.

(b) [10 points] Find the parabola $y = ax^2 + bx + c$ which best approximates the data.

Answers

(1) (a) $\lambda = 2$: $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\lambda = -1$: $\left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \right\}$

(b) YES. For instance, $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and $P = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(2) $(-4, -5)$

(3) $0, 2, -1$

(4) $\begin{bmatrix} -9 \\ 4 \\ 8 \\ 2 \end{bmatrix}$

(5) $\vec{v}_1 - 4\vec{v}_2 + \vec{v}_3 = \vec{0}$ (for instance)

(6) (a) $y = 2.2x - 1$ (b) $y = \frac{3}{7}x^2 - \frac{13}{35}x + 2$