CSUMS 03/30/09
Unsaturated Flow of Water in Soils

http://math.asu.edu/∼bdw/CSUMS/CSUMS_03-30-09.pdf

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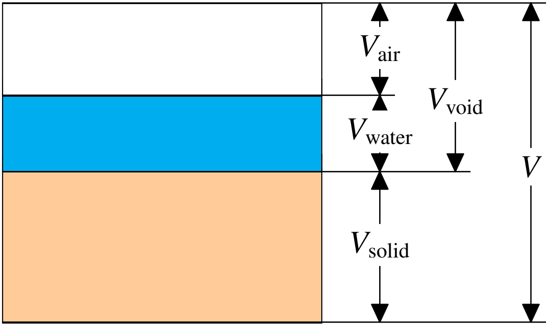
What's in a soil?

- Soil = Solid + Water + Air



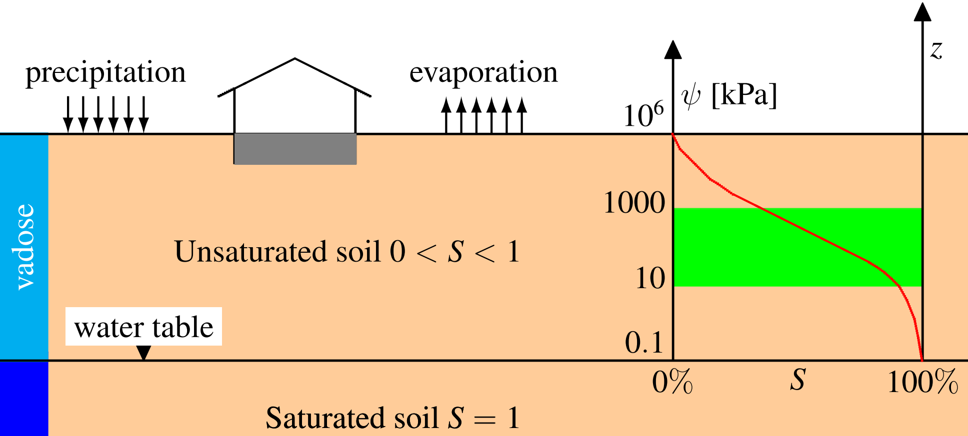
- Variables

$$\text{water content } \theta := \frac{V_{\text{water}}}{V}$$

$$\text{saturation } S := \frac{V_{\text{water}}}{V_{\text{void}}} = \frac{\theta}{\phi}$$

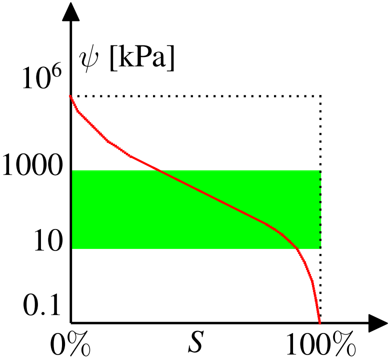
$\phi :=$ porosity

Unsaturated vs saturated

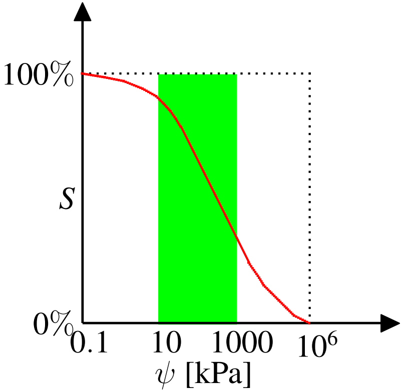


u	pore water pressure [kPa]
$h = z - \dfrac{u}{\gamma_w}$	total head [m] $\gamma_w = 9.81$ kPa/m
$\psi = \max(0, -u)$	suction [kPa]

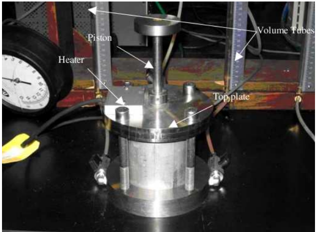
SWCC = Soil Water Characteristic Curve $S = S(\psi) \Leftrightarrow \theta = \theta(h)$

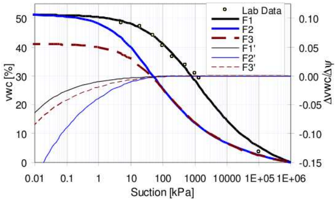


$\Rightarrow$



Source: H. Dye (2008)





Richards equation

water conservation		Darcy velocity		Richards equation
$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z}$	$+$	$q = -K(h)\frac{\partial h}{\partial z}$	$=$	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K(h)\frac{\partial h}{\partial z}\right)$

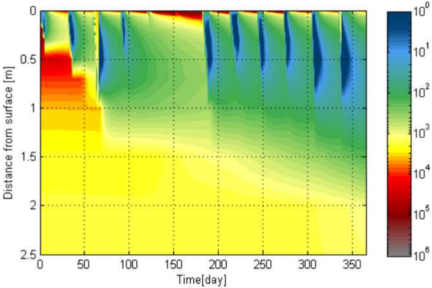
- $K(h) > 0$ is hydraulic conductivity (experimental)
- Head only h formulation = groundwater flow equation

$$\frac{\partial h}{\partial t} = \frac{1}{\theta'(h)} \frac{\partial}{\partial z}\left(K(h)\frac{\partial h}{\partial z}\right)$$

- $S'(\psi) < 0 \Rightarrow S'(h) > 0$
- nonlinear parabolic diffusion equation
- BC: $h = \text{cst}$ or $q = \text{cst}$
- Difficulties
  - $S'(h)$ and $K(h)$ only known experimentally by fitting models
  - $\theta'(h) \approx 0$ creates stiffness and numerical instabilities
  - BC switches from head to flux condition over time

SOFTWARE

- HYDRUS 1D/2D/3D by PC-Progress
  HYDRUS 1D from UC Riverside is public domain
- VADOSE/W by Geo-Slope International
- SVFLUX by SoilVision
  based on FlexPDE by PDEsolutions



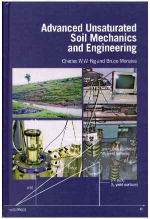
- MODFLOW, Seep/W, Unsat-H , STOMP
- Mostly adaptive FEM-based, adaptive time-stepping
- Typical simulation requires long time

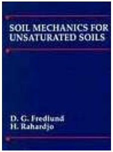
More accurate model and efficient simulation has big $$ impact

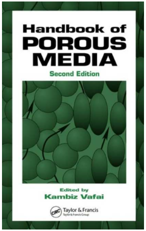
NSF Collaborative Research (with CESE)

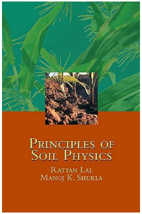
- In-house software needed to study clay soils in arid climate
- Comparative study of implicit time-stepping strategies
- Numerical modeling of boundary conditions switching
- Comparative evaluation of formulations (h or ψ-based, mixed)
- Parameter estimation from soil suction data
- Sensitivity analysis of head results w.r.t. various parameters
- Experiments with FlexPDE and HYDRUS 1D on cracked soils
- Numerical modeling of multilevel crack patterns
- Application of ADI type methods to 2D problem

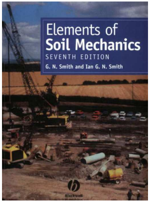
Literature

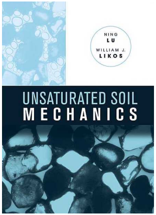












- papers on Blackboard...