Linear Programming

Lecture 17

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Column player

If CP uses mixed strategy y then he will lose no more than

$$\max_x xAy$$

where the maximum is taken over all stochastic vectors x.

Fact 1

$$\max_x xAy = \max_i \sum_{j=1}^{n} a_{ij} x_i.$$

CP Problem

CP wants:

minimize $\max_i \sum_{j=1}^{n} a_{ij} y_j$

subject to: $\sum_{j=1}^{n} y_j = 1, i = 1, \ldots, m$

$$y_j \geq 0, j = 1, \ldots, n$$

which is equivalent to

minimize w

subject to: $w - \sum_{j=1}^{n} a_{ij} y_j \leq 0, i = 1, \ldots, m$

$$\sum_{j=1}^{n} y_j = 1$$

$$y_j \geq 0, j = 1, \ldots, n$$

Minimax Theorem

Theorem 2 *For every m by n matrix A there is a stochastic vector x^* of length m and stochastic vector y^* of length n such that*

$$\min_y x^* A y = \max_x x A y^*.$$