Linear Programming

Lecture 12

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Short example

maximize $x_1 + 2x_2 + x_3$
subject to:

$$x_1 + x_3 \leq 2$$
$$x_1 + x_2 \leq 3$$
$$x_2 + x_3 \leq 1$$
$$x_1, x_2, x_3 \geq 0$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$c = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}$$

First iteration

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $c_N - c_B B^{-1} A_N = \left[\, 1 \;\; 2 \;\; 1 \,\right]$.
- entering is x_2
- $d = B^{-1} a = \left[\, 0 \;\; 1 \;\; 1 \,\right]$.
- Find the largest t such that:
  $2 - t \cdot 0 \geq 0$,
  $3 - t \cdot 1 \geq 0$,
  $1 - t \cdot 1 \geq 0$.

First iteration

- $t = 1$ and leaving variable is x_6
- New solution:

$$x^* = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}.$$

Second Iteration

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- $c_B B^{-1} A_N = \begin{bmatrix} 0 & 2 & 2 \end{bmatrix}$.
- Thus $c_N - c_B B^{-1} A_N = \begin{bmatrix} 1 & -1 & -2 \end{bmatrix}$.
- Entering variable is x_1.
- $d = B^{-1} a = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$.
- Find the largest t such that:
  $2 - t \cdot 1 \geq 0,$
  $2 - t \cdot 1 \geq 0,$
  $1 - t \cdot 0 \geq 0.$

Second Iteration

- $t = 2$ and leaving variable is x_5
- New solution:

$$x^* = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

Third iteration

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- $c_B B^{-1} A_N = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$.
- Thus $c_N - c_B B^{-1} A_N = \begin{bmatrix} 0 & -1 & -1 \end{bmatrix}$.
- Current solution is optimal.