Linear Programming

Lecture 11

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The revised simplex method

Let A be am m by $m + n$ matrix of a_{ij}'s (recall $\sum_{j=1}^{n} a_{ij}x_j \leq b_i$) with and appended identity matrix of dimension m by m (which corresponds to slack variables). Let $b = [b_1, \ldots, b_m]^T$ and let $x = [x_1, \ldots, x_{n+m}]^T$ ($x_{n+1}, \ldots, x_{n+m}$ are slack variables). Finally let $c = [c_1, \ldots, c_n, 0, \ldots, 0]$ be a vector with $m + n$ entries. Then the LP problem can be expressed as follows:

maximize cx

subject to: $Ax = b$

$$x \geq 0$$

Basic Matrix

Every basic solution x^* partitions the set of variables into a set of m basic variables and a set of n nonbasic variables. Let B (A_N) be a matrix obtained from A by considering only columns that correspond to basic (nonbasic) variables.

B is called a *basic matrix*.
In a similar way define x_B, x_N and c_B, c_N.
Fact 1 *Matrix B is nonsingular.*

Dictionary

We have:
$x_B = B^{-1}b - B^{-1}A_N x_N$
$z = c_B B^{-1}b + (c_N - c_B B^{-1}A_N)x_N$
i.e. we can easily describe a dictionary using only the knowledge of what are the basic variables (and of course original matrix A, b, and c).

Question: How to obtain a new solution from an old solution?

Revised simplex algorithm

- Solve system $yB = c_B$
- Choose an entering column (any column a of A_N such that ya is less than the corresponding coordinate of c_N). If there are none then the current solution is optimal.
- Solve $Bd = a$
- Find the largest t such that $x^*_B - td \geq 0$. If there is no such t then the problem is unbounded. Otherwise at least one component of $x^*_B - td$ equals zero and the corresponding variable is the leaving variable.

Revised simplex algorithm

- Set the value of the entering variable to t and replace the other values of x_B^* by $x_B^* - td$. Replace the leaving column of B by the entering column.