Linear Programming

Lecture 10

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The slackness theorem

Theorem 1 *Let* $x_1^*, \ldots, x_n^*$ *be a feasible solution to the primal and* $y_1^*, \ldots, y_m^*$ *be a feasible solution to the dual. Necessary and sufficient conditions for simultaneous optimality of* $x_1^*, \ldots x_n^*$ *and* $y_1^*, \ldots, y_n^*$ *are*

- $\sum_{i=1}^{m} a_{ij} y_i^* = c_j$ *or* $x_j^* = 0, j = 1, \ldots, n$
- $\sum_{j=1}^{n} a_{ij} x_j^* = b_i$ *or* $y_i^* = 0, i = 1, \ldots, m$

Another version

Theorem 2 *A feasible solution* $x_1^*, \ldots, x_n^*$ *of the primal is optimal if and only if there exist numbers* $y_1^*, \ldots, y_m^*$ *such that*

1. - *if* $x_j^* > 0$ *then* $\sum_{i=1}^m a_{ij} y_i^* = c_j$
   - *if* $\sum_{j=1}^n a_{ij} x_j^* < b_i$ *then* $y_i^* = 0$
2. - $\sum_{i=1}^m a_{ij} y_i^* \geq c_j,\ j = 1, \ldots, n$
   - $y_i^* \geq 0\ ,\ i = 1, \ldots, m$

Unique solution of (1)

Theorem 3 *If* $x_1^*, \ldots, x_n^*$ *is a nondegenerate basic feasible solution of the primal then the system in (1) of Theorem 2 has a unique solution.*