

HMMs and mapping problems

1. Consider the following Hidden Markov Model. There are two types of coin: Type 1 - fair coin, Type 2 - probability of heads is 0.2. One switches from Type 1 to Type 2 with probability 0.2 and from Type 2 to Type 1 with probability 0.4 and one stops playing (no matter what type is used) with probability 0.1. The initial probability is $p(T1) = 0.3, p(T2) = 0.7$.
 - Describe the HMM model.
 - Use Viterbi's algorithm to find probability of the most probable path for string HTHHHT.
 - Use the forward algorithm to find $P(HTHH)$.
2. We have a Markov Chain (or HMM) with k states that generates strings over some alphabet. Describe an efficient method that computes the probability that a string of length l is generated. *Hint: Modify the forward algorithm.*
3. In the STS content mapping the following data are obtained:

*	1	2	3	4	5
c_1	1	1	0	0	0
c_2	1	0	0	1	1
c_3	0	0	1	1	0
c_4	0	0	0	1	1
c_5	1	0	0	0	1

- Draw the auxiliary graph from Phase 1 of the algorithm.
 - Solve Phase 1.
 - Draw the auxiliary di-graph from Phase 2 of the algorithm.
 - Solve Phase 2.
4. In the radiation-hybrid mapping the following data was obtained.

*	1	2	3	4	5
f_1	0	1	0	1	1
f_2	1	0	0	0	1
f_3	1	0	1	1	0
f_4	0	0	0	1	1
f_5	1	0	0	0	1

- Draw the auxiliary graph.
 - Find the optimal TS cycle.
 - Use the neighborhood heuristic to solve the problem.
5. Show that there exists a graph on n vertices in which the neighborhood heuristic finds a cycle of weight larger than $f(n)OPT$ where OPT is optimal and $f(n) \rightarrow \infty$ as $n \rightarrow \infty$.
6. (*) Consider a complete graph on n vertices. What is the number of Hamiltonian cycles in this graph?