Review for Test 1, 07/17/07

July 16, 2007

1. Logical operators.
   - conjunction, disjunction, biconditional, exclusive or
   - implication: Recall that we can express $p \longrightarrow q$ also as p is sufficient for q, q is necessary for p.
2. Propositional equivalence.
3. Quantifiers.
   - Checking the truth value. For example $P(x, y)$="$x + y = 5$", universe of discourse for x and for y is the set of real numbers. Check the truth values of $\forall_x \exists_y P(x, y)$, $\exists_y \forall_x P(x, y)$.
   - Negations of quantifiers
   - Translations(1) : $F(x, y)$="x can fool y". Translate statements of the form "Everybody can fool somebody", "Mary can fool exactly one person", "Mary can fool exactly two people."
   - Translations(2) :"$B(x)$="x is a boy", $P(x, y)$="x knows y". Universe of discourse is the set of all people. Translate statements like "Every boy knows Mary", "Some boy knows Mary".
4. Methods of Proof.
   - Direct and indirect proofs of implication. Example: Show that if $n^3 + 5$ is odd and n is an integer then n is even.
   - Proof by contradiction. Show that $\sqrt{2}$ is irrational.
5. Sets.

- It is important to understand notation well. For example let $A = \{a, \{a\}, \{a, b\}\}$. Is $\{a\} \subseteq A$? Is $\{a\} \in A$?
- Cardinality of sets.
- Power set, $P(S)$ is the set of all subsets of S. For a finite set S, we have $|P(S)| = 2^{|S|}$.
- operations on sets: union, intersection, complement, and so on.
- Generalized unions and intersections: $\bigcup_{i=1}^{n} A_i$, $\bigcap_{i=1}^{n} A_i$.

6. Functions.

- Classification: injective, surjective, bijective. Checking that a given function is injective (surjective, bijective).
- Strictly increasing and decreasing functions
- Inverse function and composition of functions.
- Properties of ceiling and floor functions.

7. Sequences. The arithmetic and geometric progressions.

8. Algorithms.

- Searching: the linear search, the binary search
- Sorting: the bubble sort, the insertion sort.

9. O-notation:

- Showing that $f(x) = O(g(x))$ using C and k.
- Showing that $f(x)$ is not $O(g(x))$.
- O-estimate for various functions: polynomials, rational functions, logarithms, exponential function, factorial function, the sums.