

# Review for Final Exam, 12/13, 10:00-11:50

December 7, 2006

## 1. Echelon forms

- Elementary row operations.
- Back Substitution (free variables, leading variables).
- Gaussian elimination, Gauss-Jordan elimination.

## 2. Homogeneous systems.

## 3. Matrix algebra.

- Addition, scalar multiplication, multiplication.
- Inverse of a matrices:
  - $2 \times 2$  matrices formula.
  - Method for finding  $A^{-1}$ .

## 4. Determinants of $2 \times 2$ matrices.

- Transpose of a matrix and relation to determinants.
- Solving systems using Cramer's Rule.

## 5. Higher-order determinants.

- $ij$ th minor,  $M_{ij}$  = determinant of a submatrix obtained by deleting  $i$ th row and  $j$ th column.
- $ij$ th cofactor,  $A_{ij} = (-1)^{i+j}M_{ij}$ .

- Cofactor expansions of determinants.

$$\det A = a_{i_1}A_{i_1} + a_{i_2}A_{i_2} + \cdots + a_{i_n}A_{i_n}$$

$$\det A = a_{1j}A_{1j} + a_{2j}A_{2j} + \cdots + a_{nj}A_{nj}.$$

## 6. Determinants and the inverse of a matrix

- Properties of determinants.
- Adjoint matrix.
- Inverse of a matrix.

## 7. Vector Spaces.

- Abstract vector spaces and subspaces.
- Solution spaces.
- Linear combinations and  $\text{span}(S)$ .
- Linear independence.
- Bases of vector spaces. Finding bases of solution spaces.

## 8. Vector spaces associated with a matrix $A$ .

- Row space. Finding a basis of  $\text{Row}(A)$ .
- Column space. Finding a basis of  $\text{Col}(A)$ .
- Null space. Finding a basis of  $\text{Null}(A)$ .
- Rank of a matrix:  $\text{rank}(A) + \dim(\text{Null}(A)) = n$ .

## 9. Orthogonal vectors.

- Scalar product, length of a vector, Cauchy-Schwarz inequality.
- Orthogonal and orthonormal bases.
- $V^\perp$  and properties.
- $\text{Row}(A)^\perp = \text{Null}(A)$ ,  $\text{Null}(A)^\perp = \text{Row}(A)$ .
- Finding a basis of  $V^\perp$ .

## 10. Least squares solution. Projections on a subspace.

- Finding the least squares solution of  $Ax = b$ .
- Finding a projection of vector  $b$  on  $V$ .
- Finding a projection of  $b$  on  $V$  in the case we have an orthogonal basis of  $V$ .
- Finding an orthogonal basis of  $V$ . Finding an orthonormal basis of  $V$ .

11. Eigenvalues and eigenvectors.

- Finding eigenvalues and eigenvectors.
- Diagonalization.