Mat 342 Review for Final Exam

Part 1, Computations

- **System of Linear Equations and Gaussian elimination**
- **Inverse of a matrix and singular matrices**
- **Finding the determinant of a matrix**
- **Finding the adjoint of a matrix**
- **Cramer's formulae**
- **Checking if a vectors are linearly independent**
- **Checking if a vectors form a spanning set**
- **Finding a basis and dimension of a vector space** $span(S)$
  S is a set of vectors in R^n, matrices, polynomials, functions.
- **Finding the transition matrix**
  - In R^n, $S = U^{-1}V$.
  - If $V = P_n$ or $C[a, b]$ set up the system of equations for coefficients.
- **Finding a basis and dimension of** $N(A), R(A), R(A^T)$
- **Linear Transformations**
  - Checking if a given function is a linear transformation.
  - Finding the kernel.
- **Matrix representation of a linear transformation**
- **Finding a basis and dimension of** $span(S)^{\perp}$
- **The least squares solutions and the normal equation**
- **Checking if a given function defines an inner product**
- **Checking if a given function defines a norm**

- **Orthogonality**
  - Checking if vectors (polynomials, functions) are orthogonal or orthonormal.
  - Gram-Schmidt procedure for finding an orthonormal basis.
- **Eigenvalues**
  - Finding eigenvalues, eigenspaces.
  - Finding the sum and the product of all eigenvalues.
  - Diagonalization of a matrix.

Part 2, Theory

- **Singular matrices**
- **Equivalent conditions for an $n \times n$ matrix to be nonsingular**
- **Definition of the null space of a matrix**
- **Determinants**
  - Definition of the determinant (Laplace expansion)
  - Basic properties of the determinant: elementary matrices, the transpose of a matrix.
  - If A, B are $n \times n$ then $det(AB) = det(A)det(B)$.
- **Definition of the adjoint of a matrix**
- **Linearly independent vectors and spanning sets**
  - Definitions.
- **A basis and dimension**
  - Definitions.
  - Facts about spanning sets and independent sets when the size of the set is equal to the dimension of the vector space.
- **Row space, column space, null space**

- Definitions. The rank and the nullity of a matrix.
- The rank-nullity theorem.
- Correspondence between row spaces (column spaces) of equivalent matrices.
- The dimension of the column space in relation to the dimension of the row space.
- The null space is the orthogonal complement of the row space. The null space of the transpose is the orthogonal complement of the column space.

- **Linear Transformations**
  - Definition of a linear transformation.
  - Every linear transformation L from R^n to R^m has the form $L(x) = Ax$
  - The kernel and the image.
  - Similarity of matrices.

- **Orthogonality**
  - Orthogonality of vectors. Orthogonal subspaces. The orthogonal complement of a subspace. The set of orthogonal and orthonormal vectors.
  - Properties of the orthogonal complement of S.

- **Inner product spaces and normed vector spaces**
  - Definition of an inner product. Definition of a norm.
  - Important examples of inner products and norms.
  - Orthonormal basis.

- **Eigenvalues**
  - Definitions of eigenvalues, eigenvectors, eigenspaces.
  - Properties of eigenvectors, trace.
  - Diagonalization.