
Distributed packing algorithms for planar graphs

A. Czygrinow

Department of Mathematics & Statistics

Arizona State University

M. Hańćkowiak, W. Wawrzyniak

Faculty of Mathematics and Computer Science

Adam Mickiewicz University

Poznań, Poland

Distributed model

- 1 Message-passing model.
- 2 Synchronized.
- 3 Unique identifiers.
- 4 No restriction on the amount of local computations.
- 5 No restriction on the length of messages sent.

Want

- 1 **Deterministic algorithms.**
- 2 **Time complexity:** The number of rounds is **poly-logarithmic** in the order of a graph.

Notation

- $|G|$ is the order of graph G , $\|G\|$ denotes the size of G .

Packing problem

- **Packing of H in G** is a set of pairwise-disjoint subgraphs of G , each isomorphic to H .
- $\nu_H(G)$ - the packing number of H in G .
- **In this talk $|H|$ is constant** but **the result holds for $|H| = \text{polylog}(|G|)$.**

Examples

- $\nu_{K_2}(G)$ is the size of the max matching.
- If $|C| = |G|$ where C is a cycle then $\nu_C(G) = 1$ when G is Hamiltonian.

Result

Theorem 1 *Let k be a fixed constant. There is a distributed algorithm which given H and k returns a packing of H in G with value of at least*

$$(1 - O(|H|/\log^k |G|))\nu_H(G).$$

The algorithm runs in a $\text{polylog}(|G|)$ number of rounds provided $|H| = \text{polylog}(|G|)$ and $k = O(1)$.

Related results

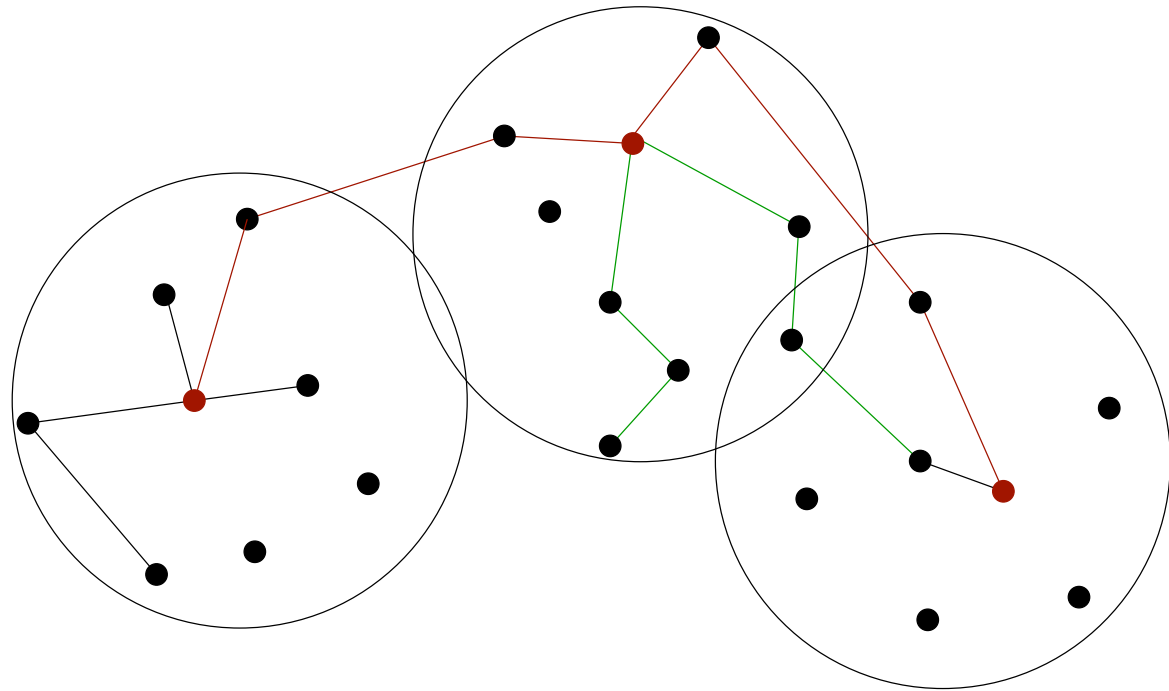
- **Different models and randomized algorithms:** Not in this talk.
- **Planar graphs:** Approximations of MaxIS , MaxMatching , MDS (ACMH, 2007)
- **Planar graphs:** Constant approximation of MDS in $O(1)$ rounds (previous talk if you were not here)
- **Different families of graphs:** Unit-disk graphs (F. Kuhn et. al. 2005-2008, ACMH 2007)

Partitions of graphs

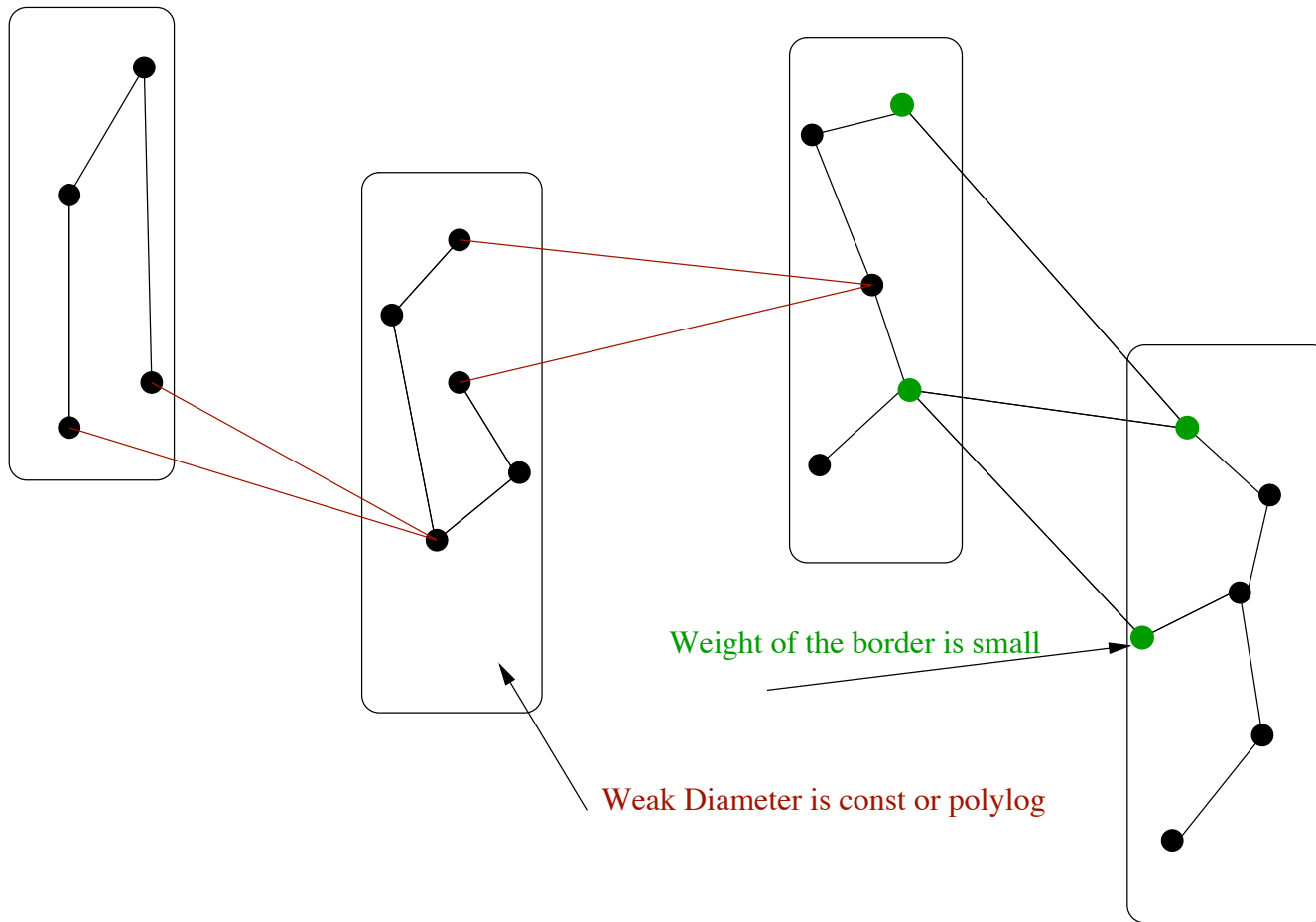
- **An (a, b) -ruling set in G :** Set D such that
 - $dist_G(d, d') \geq a$ when $d, d' \in D$ and
 - for every x there is $d \in D$ with $dist_G(x, d) \leq b$.

**An $(k, O(k \log |G|))$ -ruling set can be found in $O(k \log \log |G|)$.
(Awerbuch, Goldberg, Luby, Plotkin 1989)**

- **Ruling set gives a ruling set partition of G**



- **An (α, β) -partition of a planar graph G with weights $\omega : V(G) \rightarrow \mathbb{R}^+ \cup \{0\}$:** Partition (P_1, \dots, P_k) of $V(G)$ such that
 - $\sum \omega(\partial(P_i)) \leq \beta \omega(G)$ and
 - $G[P_i]$ is connected and the **weak diameter** of $G[P_i]$ is at most α .
- **$O(O(1), 1/2)$ -partition and $O(\text{polylog}|G|, 1/\log^{O(1)}|G|)$ -partition can be found in $\text{polylog}|G|$ rounds.**

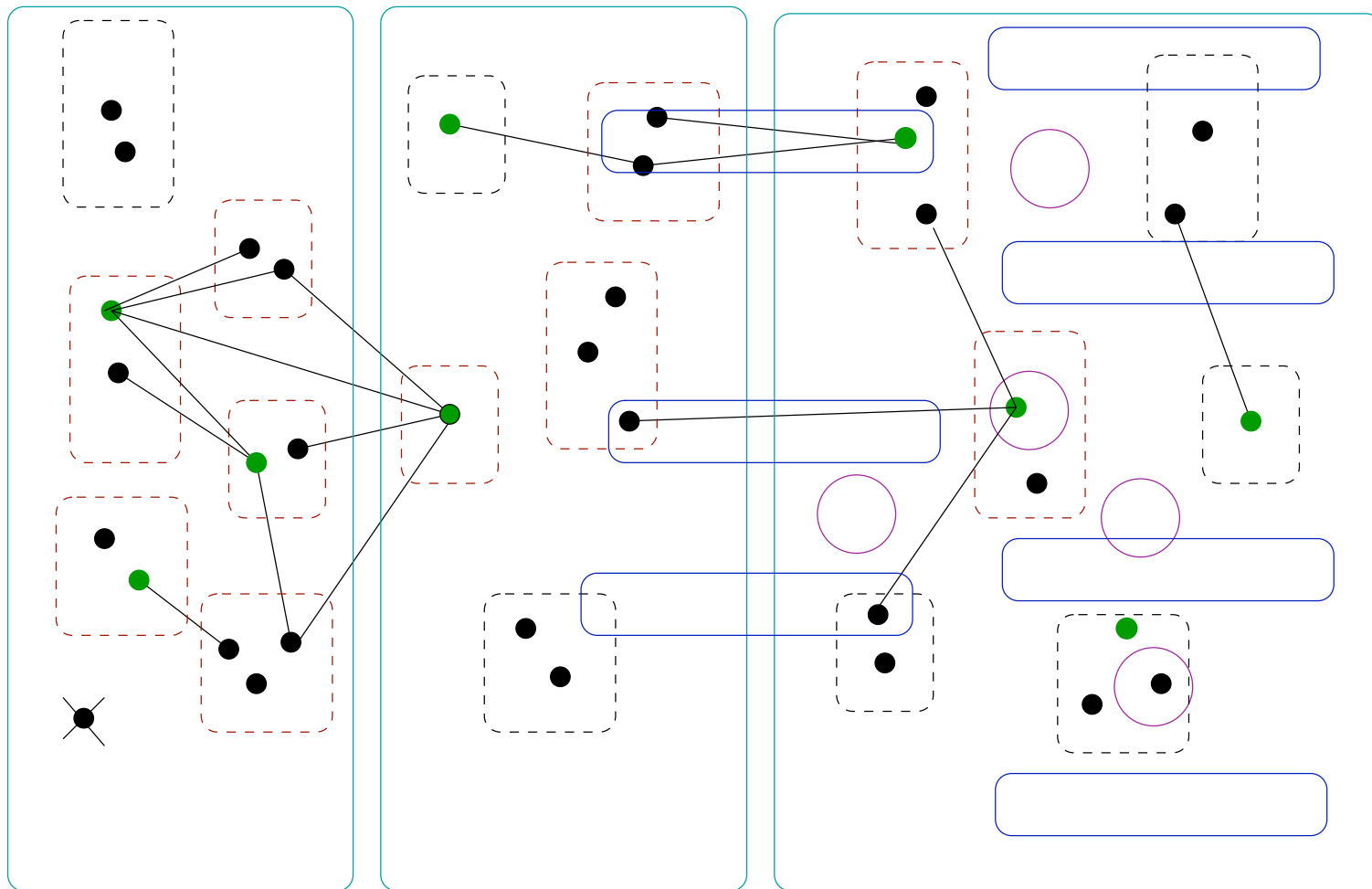


Algorithm

Packing of H in G . Given: k .

- 1 Vertex checks if there is a copy of H that contains it. If not then delete.
- 2 Find a "special" partition $B = (B_1, \dots, B_q)$ of $V(G)$ and a "small" set Z which is a vertex cover of $G[\partial(B)]$. (**How?**)
- 3 Contract B_i to b_i and set $\omega(b_i) = |B_i \cap Z|$. (**Graph G'**)
- 4 Find a $O(\text{polylog}(|G|), 1/\log^{k+1}(|G|))$ -partition $A = (A_1, \dots, A_p)$ of the graph from step 3.

5 Find **optimal solution** in each of $G[A_i]$'s and return the union.



H

Optimal H

How does the argument go?

- Let \mathcal{H} be an optimal packing and let $\mathcal{H}_{IN} \subseteq \mathcal{H}$ consists of only these copies which are contained in one of A_i 's.
- We find a packing which is better than $|\mathcal{H}_{IN}|$.
- What do we know about $\mathcal{H} \setminus \mathcal{H}_{IN}$?
 - At most the size of the maximum matching in $G[\partial(A)]$ which is at most the number of vertices in Z that are in $\partial(A)$...

– ... which is at most

$$\omega(\partial(G')) \leq |Z| / \log^{k+1}(|G|)$$

– and Z is such that

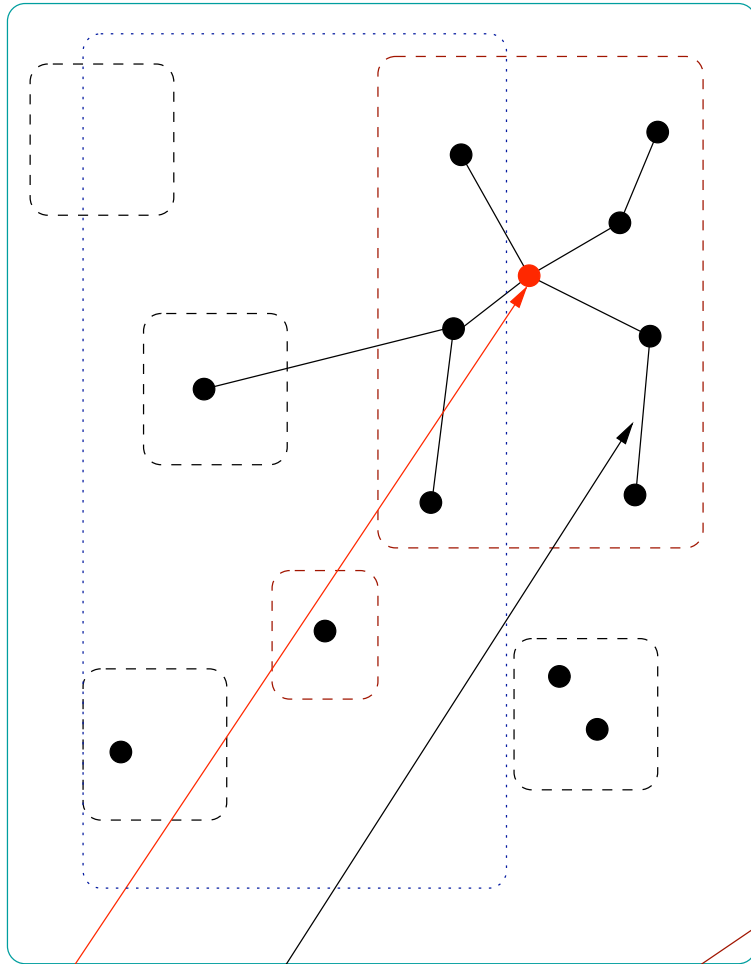
$$|Z| \leq \nu_H(G) \log(|G|).$$

Finding Z and "special" partition B

- Find a $(2c + 1, O(c \log |G|))$ -ruling set D in G .
- Iterate:
 - (a) Find a **ruling set partition** around D and contract partition classes.
 - (b) Find a $(O(1), 1/2)$ -partition $P = (P_1, \dots, P_s)$ of the graph from (a). (No weights.)
 - **Separate** each P_i from the rest of the graph by set Z_i .
(How?)

How does the argument go?

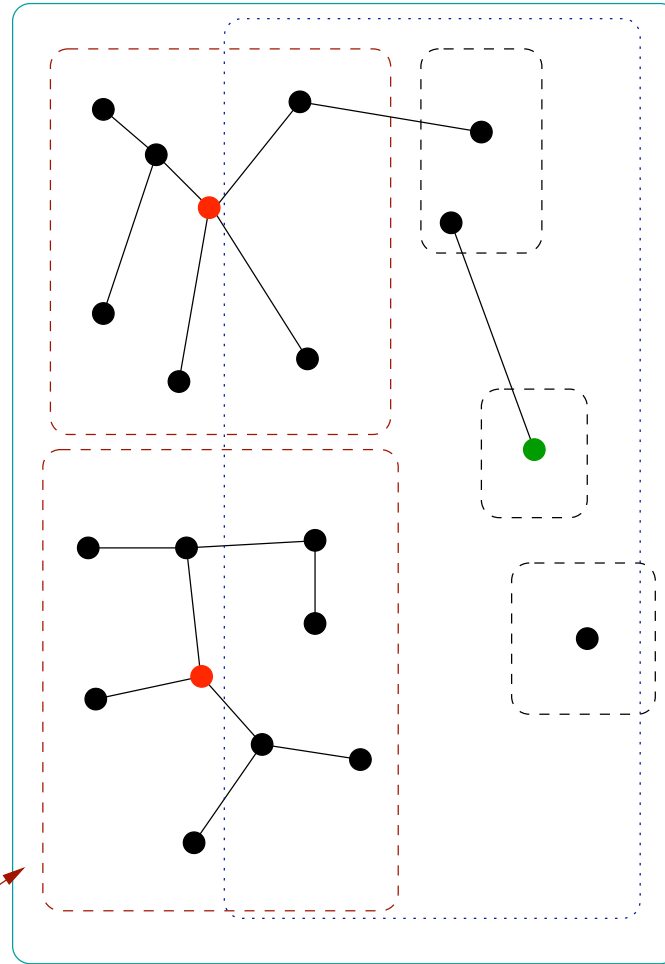
- We know that $|D| \leq \nu_H(G)$.
- We know that the number of border clusters in P is at most $|D|/2$.
- We need to separate them using $O((\# \text{border clusters}) \cdot \text{diam}(\text{max ruling set partition cluster}))$ vertices.
- We must argue that after an iteration the set D has similar properties.



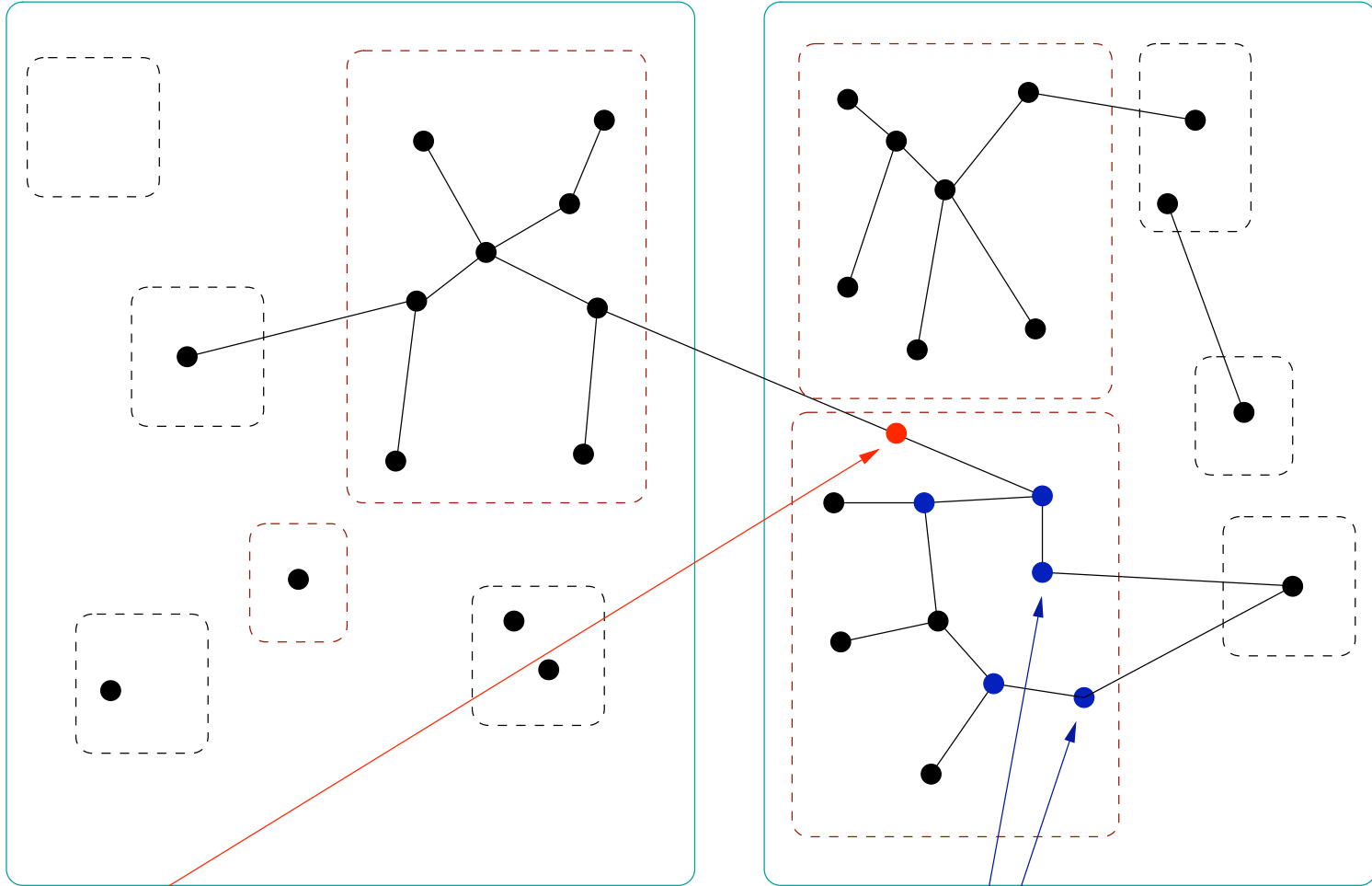
D

BSF

diam is $O(\log |G|)$

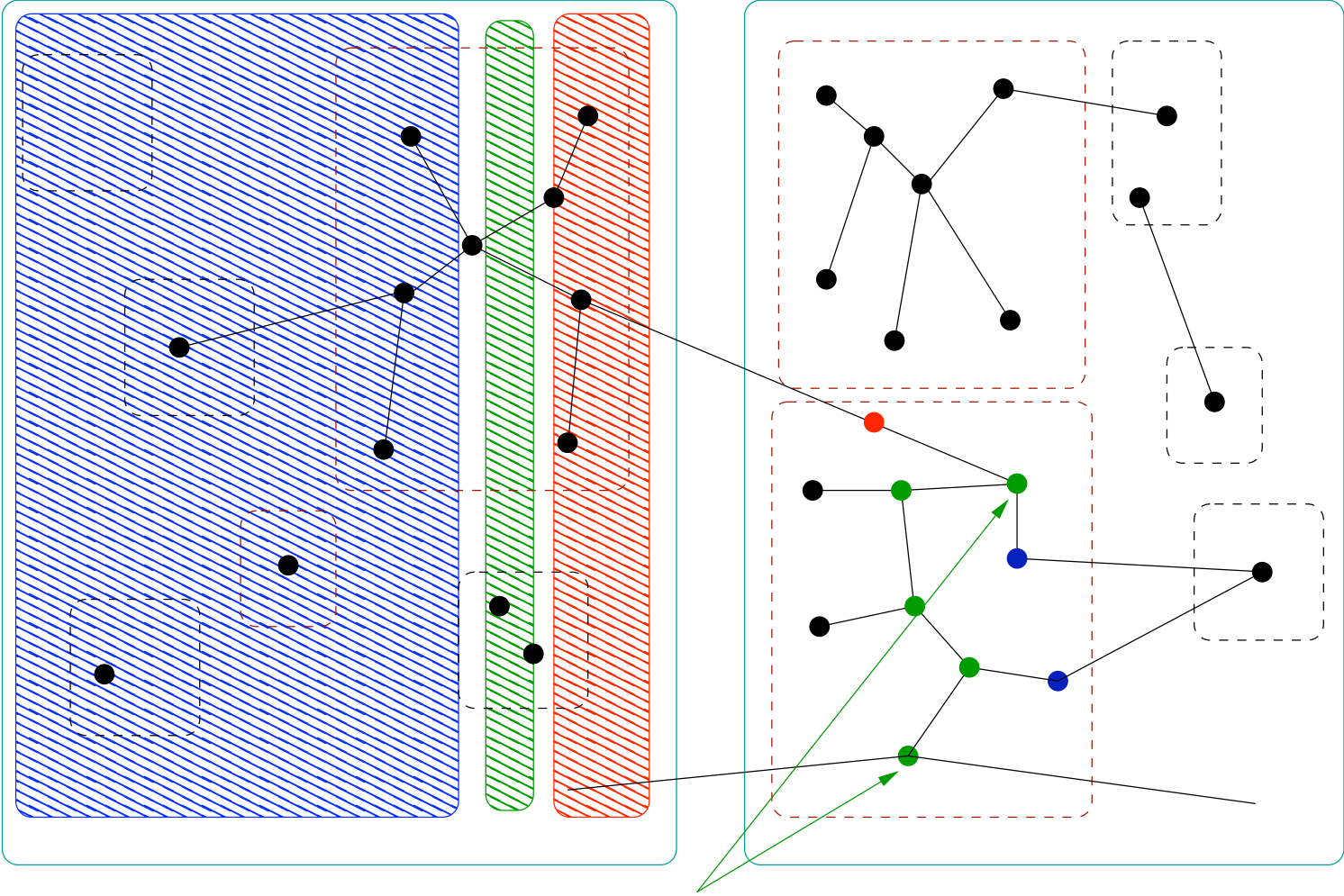


Want: Separate Inside from the rest



red vertices with neighbors outside

blue vertices with neighbors inside



Green vertices

Conclusions

- Deterministic, distributed approximation algorithm for the vertex-packing problem in planar graphs.