Distributed almost exact approximations for minor-closed families

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Computational models

- Sequential
- Parallel
- **Distributed**

Example: The Maximal Independent Set Problem.

Distributed model

1. **Network** is modeled as an **undirected graph**.
2. **Global clock**. In a single round a vertex can send, receive, and compute.
3. Unique identifiers.
4. The amount of local computations can be **unlimited**.

Goals

- **Deterministic** algorithms.

- The number of rounds is **poly-logarithmic** in the order of a graph - **efficient algorithm**. [N. Linial]

Coloring a cycle C

- A trivial algorithm finds a coloring C with $\chi(C)$ colors in $O(|C|)$ **rounds**.

- Algorithm of Cole-Vishkin finds a coloring with $O(1)$ colors in $O(\log^* |C|)$ **rounds**.

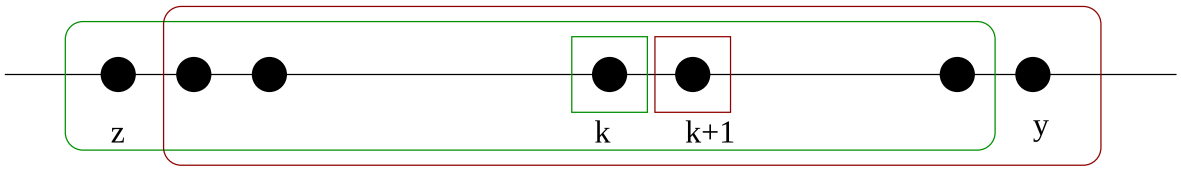
- Linial [1992]: No faster algorithm exists.

Linial's argument

- Say k is the number of rounds and identifiers come from $\{1, \ldots, |C|\}$.

- Each vertex x knows a vector of length $2k + 1$.

- An algorithm is just a coloring of these vectors so that $(x_1, \ldots, x_{2k}, y), (z, x_1, \ldots, x_{2k})$ receive different colors.



- Now find a lower bound for the chromatic number of this auxiliary graph ($\Omega(\log^{(2t)} |C|)$).

Minor-closed families of graphs

- X is called a **minor** of Y if it can be obtained from a subgraph of Y be a sequence of edge contractions.

  Note: Equivalent to contracting connected subgraphs.

- $\mathcal{C}$ is **minor-closed** if for every $G \in \mathcal{C}$, any minor of G is in $\mathcal{C}$.

- J. Nesetril, P. Ossona de Mendez [2005, 2006]

$$\rho_{\mathcal{C}} = \sup_{\mathbf{G} \in \mathcal{C}} \frac{||\mathbf{G}||}{|\mathbf{G}|}.$$

  $\mathcal{C}$ is a proper minor-closed family iff $\rho_{\mathcal{C}} < \infty$.

Results (**Joined work with M. Hańćkowiak** 2007)

Efficient distributed approximations for:

- the maximum weight **matching problem** in minor-closed families.
- the minimum weight **dominating set problem** in minor-closed families.

Approximation error: $\left(1 \pm \frac{1}{\ln^k |G|}\right)$.

Related Work

- **Different models:** A lot ...

- **Randomized algorithms:** (1) L. Jia, R. Rajaraman, and R. Suel, (2) D. Dubhashi, A. Mei, A. Panconesi, J. Radhakrishnan, A. Srinivasan, (3) F. Kuhn, R. Wattenhofer

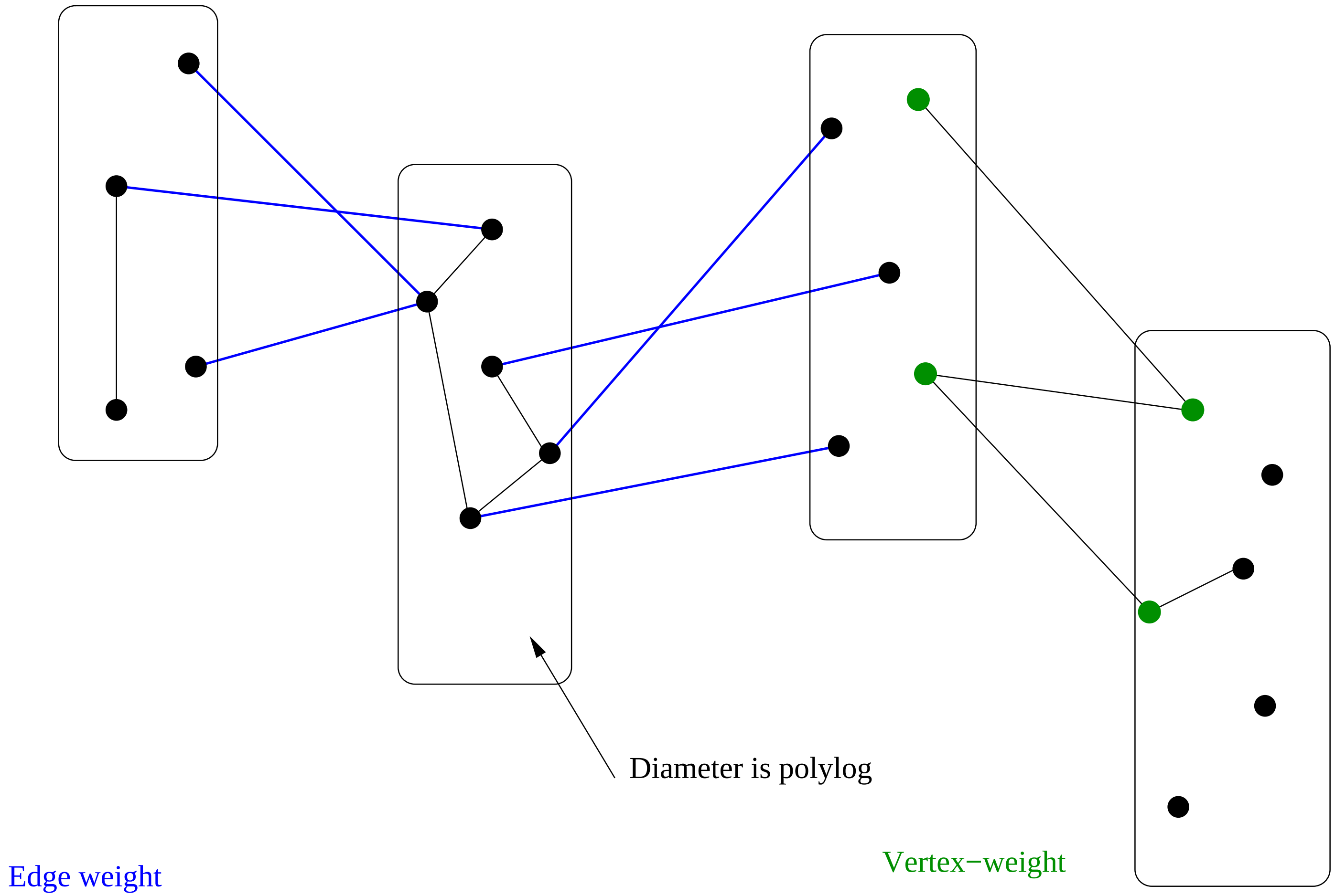
- **Different classes of graphs:** UDG (Kuhn et. al., CH)

- **Planar graphs:** Approximations for MaxIS (planar), Max-Matching (planar), MDS (outerplanar) (CH, 02), unweighted minor-closed (CH, 06).

Weighted graphs

- Weighted graphs (G, ω) with $\omega : V(G) \rightarrow N_0$, $(G, \bar{\omega})$ with $\bar{\omega} : V(G) \rightarrow N_0$

- **Idea:** Find a **special** partition $\mathcal{P}$ of G so that $\omega(\partial\mathcal{P}))$ or $\bar{\omega}(\partial\mathcal{P}))$ is small.


Diameter is polylog
Edge weight
Vertex−weight

Algorithm for edge-weighted graphs

- Let $\mathcal{C}_\rho$ be a proper minor-closed family of graphs. There is an efficient distributed algorithm which given $k \geq 1$ finds in an edge-weighted graph $(G, \bar{\omega})$ with $G \in \mathcal{C}_\rho$ a partition $\mathcal{P} = (V_1, \ldots, V_l)$ with $diam(G[V_i]) = poly - log(|G|)$ and such that

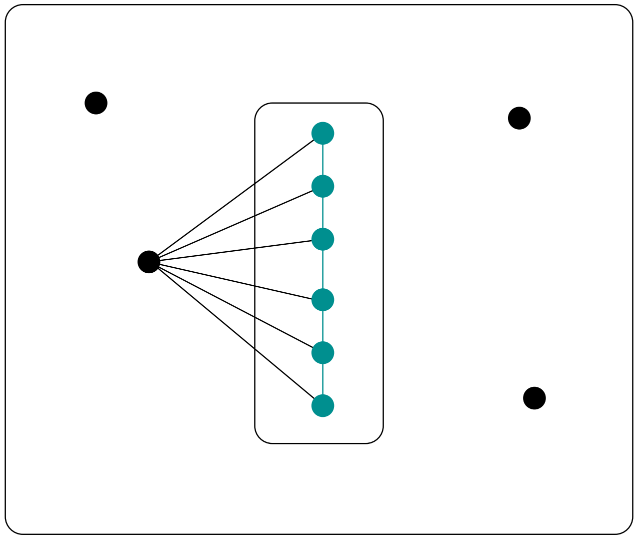
$$\bar{\omega}(\partial \mathcal{P}) \leq \bar{\omega}(G) / \ln^k |G|.$$

(CH, 06)

Diameters

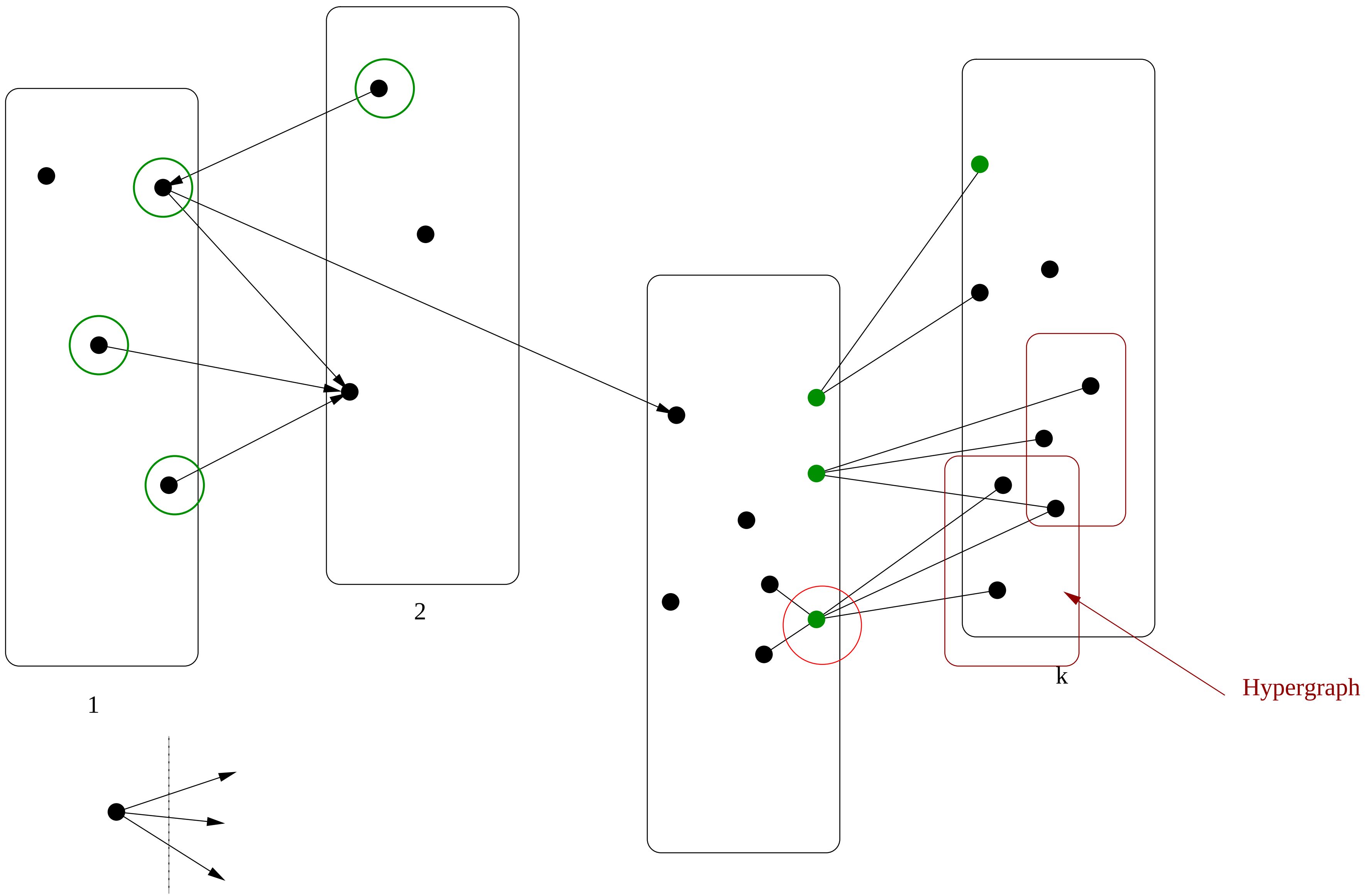
- **Strong diameter** of a subgraph and **weak diameter** of a subgraph.

- $weak \leq strong$.



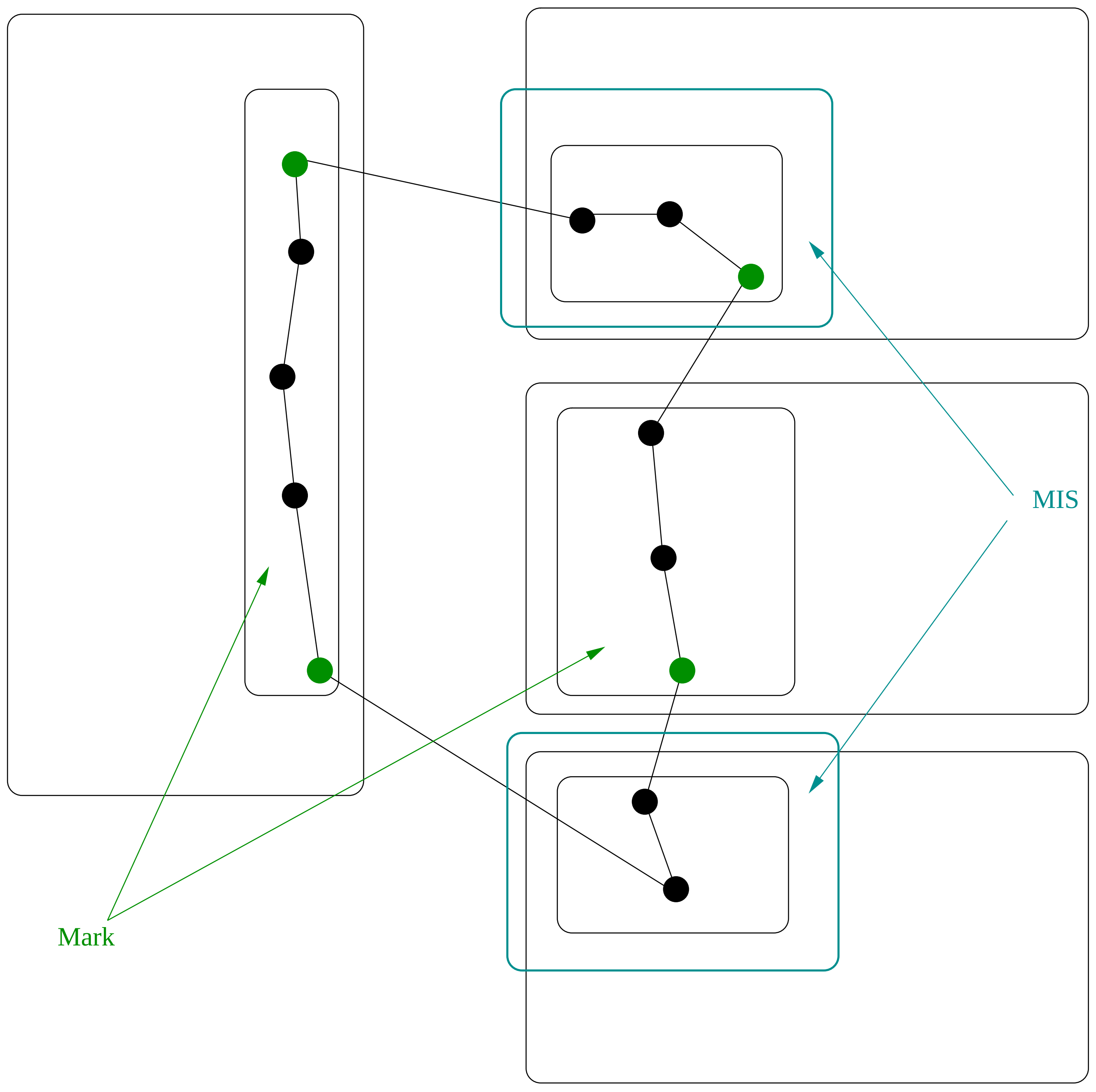
Outline of the partitioning algorithm

- Phase 1
  - Find an orientation, auxiliary edge weights, and an **edge-weight partition**.
  - Mark some of the border vertices.
  - Consider an **auxiliary hypergraph**, find **(a heavy) IS** in it, and mark all but the vertices in **IS**.



2
1
k
Hypergraph

- Phase 2
  - Look at the **auxiliary graph** obtained by contracting components consisting of unmarked vertices.
  - Find a **heavy MIS** I in this graph and mark everybody not in I.



MIS
Mark

Outline of the partitioning algorithm

- After finding subgraphs **contract them to vertices**. Recompute the weight (new weight = the weight of marked vertices in a component) and...

- ...iterate.

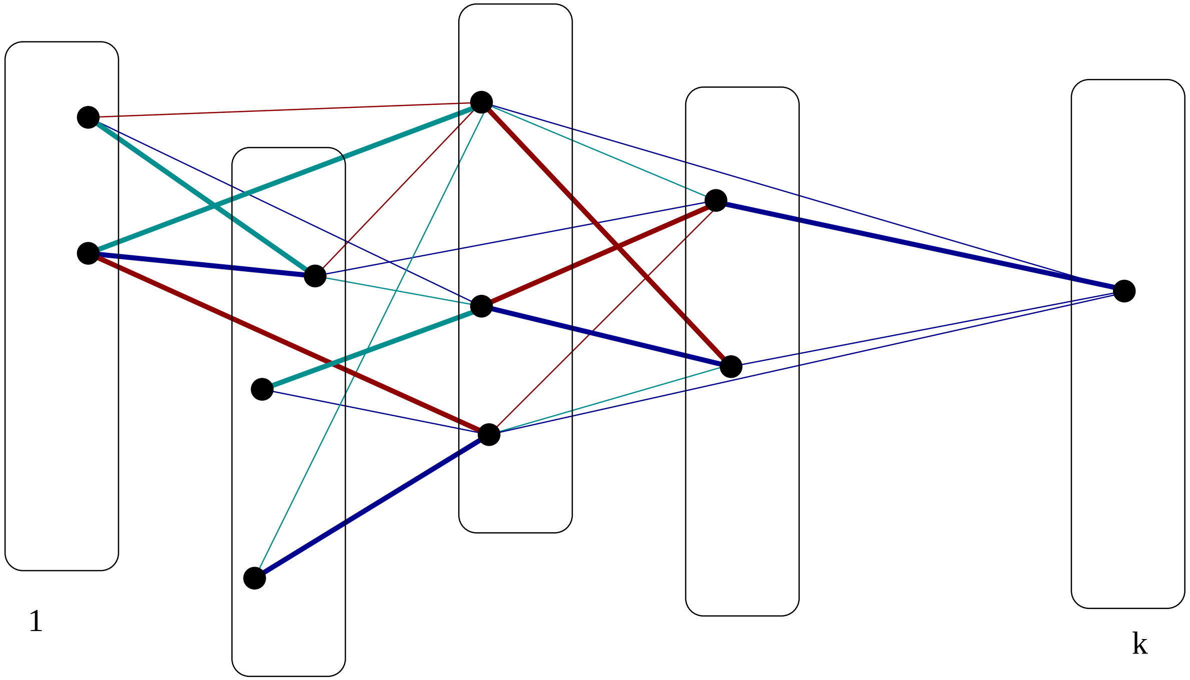
Applications

- Efficient distributed approximation for the maximum weight matching problem.

- Efficient distributed approximation for the minimum weight dominating set problem.

Matchings

1. Define weights on **vertices**.



$$\omega(v) := 0 + \sum_{(v,u)\in M_i} \bar{\omega}(v,u).$$

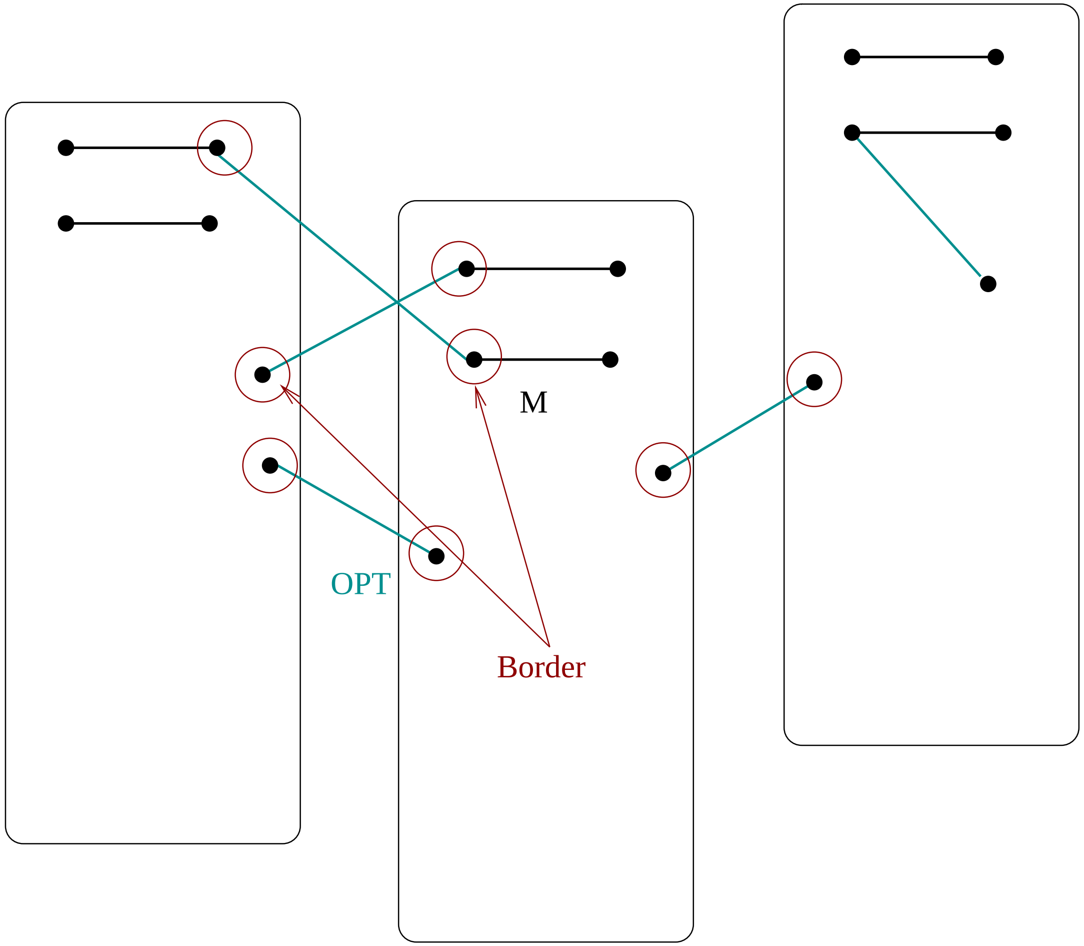
2. Note that

$$\sum_v \omega(v) = O(\beta(G))$$

and

$$\bar{\omega}(u,v) \leq \omega(u) + \omega(v).$$

3. Find a vertex-weight partition $\mathcal{P} = (V_1, \ldots, V_k)$ and an optimal solution in each of $G[V_i]$'s. **Return the union** M.


M
OPT
Border

4.

$$\beta(G) = OPT_{inside} + OPT_{border} \leq \bar{\omega}(M) + \omega(\partial\mathcal{P})$$