Practice Test#2 MAT 275 Dr. Taylor's Class Spring'08

Instructions: Give yourself one hour to work this practice exam.

1. Consider the differential equation $y'' - (1-x)y' - y = 0$.

   a) Show that $y_1 = e^x$ is a solution of this equation

   b) There is another solution of this equation which can be written in the form $y_2 = u(x)e^x$. Substitute y_2 into this differential equation to find an first order differential equation for $u'(x)$, and then solve this equation and integrate to obtain $u(x)$. Verify that $y_2 = -1 - x$.

   c) Use the Wronskian to show that your y_1, y_2 are linearly independent.

   d) Find the solution of the differential equation which satisfies $y(1) = 0, y'(1) = 1$.

2. Consider the nonhomogeneous differential equation $y'' - 4y' + 4y = e^{2x}$.

   (a) Find a particular solution using the method of undetermined coefficients.

   (b) Find a particular solution using the variation of parameters method.

   (c) Solve the initial value problem with this differential equation and the initial conditions $y(0) = 0, y'(0) = 0$.

3. A body with mass 1 kg is attached to the end of a spring that is stretched to a length of 10cm by a force of 10 Newtons. The body is also attached to a dashpot which delivers a force of magnitude 1 Newton when the body is moving at 1 m/s.

   (a) Is this system over-, critically- or under-damped?

   (b) The body is pulled $10cm$ to the right and at time $t = 0$ is released from a standstill. At what time does the body reach the equilibrium position for the first time, and how fast is it moving when it does so?

4. Solve the initial value problem $y''' - 2y'' - 5y' + 6y = 0$; $y(0) = 1, y'(0) = 0, y''(0) = -1$

5. Consider the system of differential equations

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

   a) Compute general solution of this system using substitution.

   b) Compute the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$$

   c) Use your answer to part b) to find the solution of the differential equation again using eigenvalues and eigenvectors.