

Four ways to solve a system.

Consider this 2 by 2 system:
$$\begin{aligned} 3x + 4y &= -3 \\ 2x - 5y &= 1 \end{aligned}$$

Method I: Algebraically Using Elimination.

$$\begin{aligned} -2(3x + 4y = -3) &\Rightarrow -6x - 8y = 6 \\ 3(2x - 5y = 1) &\Rightarrow 6x - 15y = 3 \end{aligned} \Rightarrow -23y = 9 \Rightarrow y = -\frac{9}{23}$$

Back substitute to get $x = -\frac{11}{23}$. Method of substitution is also an option.

Method II: Augmented Matrices and Row Operations.

$$\begin{aligned} 3x + 4y = -3 \\ 2x - 5y = 1 \end{aligned} \Rightarrow \begin{bmatrix} 3 & 4 & -3 \\ 2 & -5 & 1 \end{bmatrix} \Rightarrow R_1 = \frac{1}{3}r_1 : \begin{bmatrix} 1 & 4/3 & -1 \\ 2 & -5 & 1 \end{bmatrix} \Rightarrow R_2 = r_2 - 2r_1 : \begin{bmatrix} 1 & 4/3 & -1 \\ 0 & -23/3 & 3 \end{bmatrix} \\ \Rightarrow R_2 = -\frac{3}{23}r_2 : \begin{bmatrix} 1 & 4/3 & -1 \\ 0 & 1 & -9/23 \end{bmatrix}. \text{ The bottom line gives } y = -\frac{9}{23}, \text{ back substitute to get } x = -\frac{11}{23}.$$

Method III: Matrix algebra (the “ $A^{-1}B$ ” form).

$$\begin{aligned} 3x + 4y = -3 \\ 2x - 5y = 1 \end{aligned} \Rightarrow \begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}. \text{ Let } A = \begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix}, B = \begin{bmatrix} -3 \\ 1 \end{bmatrix}. \text{ We can write this system as a matrix equation } AX = B. \text{ By applying } A^{-1}, \text{ we get } X = A^{-1}B. \text{ Let your calculator do the calculations:}$$
$$X = A^{-1}B = \begin{bmatrix} -11/23 \\ -9/23 \end{bmatrix}.$$

Method IV: Cramer's Rule (Determinants).

The determinant Δ of a matrix is the product of the main diagonal minus the product of the secondary diagonal. Let $A = \begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix}$. The determinant Δ is $(3)(-5) - (2)(4) = -15 - 8 = -23$. To calculate x , replace

the x column in A with the constants: $\begin{bmatrix} -3 & 4 \\ 1 & -5 \end{bmatrix}$, then calculate its determinant: $(-3)(-5) - (1)(4) = 15 - 4 =$

11. Then divide by Δ to get $x = -\frac{11}{23}$. For y , replace the y column in A with the constants: $\begin{bmatrix} 3 & -3 \\ 2 & 1 \end{bmatrix}$ and

calculate the determinant: $(3)(1) - (2)(-3) = 9$, then divide by Δ to get $y = -\frac{9}{23}$.

Method V: Bribery.

“Mr. Surgent, I’ll wash and wax your truck if you give me the solution to this system.” (Warning: Mr. Surgent can’t always be trusted to tell the truth)