

## Lectures 21, Tu., October 30, 2007

### Reading homework: Chapter 6

**1. Modified Lotka-Volterra predator-prey model.** The Lotka-Volterra model is one of the earliest predator-prey models. It forms the basis of many models used today in the analysis of population dynamics. Unfortunately, in its original form Lotka-Volterra has some significant problems. The most noteworthy is that the positive equilibrium is neutrally stable. The predator and prey populations cycle endlessly without settling down. While this cycling has been observed occasionally in nature, it is far from common and when it appears the limit cycle is usually does not dependent on initial data. Hence Lotka-Volterra predator-prey model by itself may not be appropriate to model any specific predator-prey systems. Additional context specific structures must be incorporated to make a predator-prey model more relevant to nature. Much of these problems stems from the fact that the model assumes exponential growth for prey in the absence of predators. Hence a generic modification is to replace that assumption by a logistic growth. This yields the following modified Lotka-Volterra predator-prey model

$$\begin{cases} \frac{dx}{dt} &= rx(1 - x/K) - bxy, \quad x(0) > 0, \\ \frac{dy}{dt} &= e xy - cy, \quad y(0) > 0, \end{cases} \quad (1.1)$$

where  $r$  is the prey growth rate and  $K$  its carrying capacity determined by its environment.

We have rigorously shown that solution of (1.1) is positive and bounded.