11262. *Proposed by Ashay Burungale, Satara, Maharashtra, India.* In a certain town of population $2n+1$, one knows those to whom one is known. For any set A of n citizens, there is some person amongst the other $n+1$ who knows everyone in A. Show that some citizen of the town knows all the others.

Solution by Christopher Carl Heckman, Arizona State Univeristy, Tempe, AZ: The contrapositive to this statement (which is equavalent to the original statement) will be shown: If the town has $2n+1$ people, everyone knows those to whom they are known, and nobody knows everyone in the town, then there is a set A of n citizens, such that none of the other $n+1$ knows everyone in A.

First, everyone will receive a "color", either red or green, in the following manner: Order the people as $P_1, P_2, \ldots, P_{2n+1}$. For each i from 1 to $2n+1$, do the following: If person P_i has not received a color yet, choose a person $P_{p(i)}$ such that P_i does not know $P_{p(i)}$. If $P_{p(i)}$ has received a color already, assign the other color to P_i; otherwise, assign red to P_i and green to $P_{p(i)}$.

Because of the way that the colors were assigned, for each "red" person, there is a "green" person that they do not know, and vice versa.

Due to symmetry, we may assume that there are at most n red people, whom we put into set A. If the number of "red" people is $n-k$ for some $k>0$, choose k green people and add them to A.

Now the set A satisfies the claim: Everyone not in A is green, and therefore there is a red person (whom is in A) whom they do not know. Hence no one not in A knows everyone in A, as claimed.